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An Over-Complete Independent Component Analysis (ICA) Approach to Magnetic Resonance Image Analysis

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Abstract

This paper presents a new application of independent component analysis (ICA) in magnetic resonance (MR) image analysis. One of most successful applications for ICA-based approaches in MR imaging is functional MRI (fMRI) which basically deals with one-dimensional temporal signals. The ICA approach proposed in this paper is rather different and considers a set of MR images acquired by different pulse sequences as a 3-dimensional image cube and performs image analysis rather than signal analysis. One major difference between the fMRI-based ICA approaches and our proposed ICA-based image analysis is that the ICA used in the former is under-complete as opposed to the latter which uses over-complete ICA. Such a fundamental difference results in completely different applications.

I. INTRODUCTION

Independent component analysis (ICA) has found many applications in communications, signal and image processing [1]. One of its particular applications in which the ICA has generated considerable interests is functional MRI (fMRI) which is a method that provides functional information of MR images at time series as a temporal function. Since the samples collected along a temporal sequence, denoted by L is generally greater than the sources to be separated, denoted by p, the ICA used for fMRI is generally under-complete in the sense that the ICA deals with under representation of a mixed model. In this case, the ICA intends to solve an over-determined system with $L > p$ consisting L equations specified by the number of samples with signal sources to be separated as p unknowns. So, a general approach to the problems of this type is dimensional reduction (DR) such as principal components analysis (PCA) to make the mixing matrix used in the ICA a square matrix. In this paper, we take a complete opposite approach and explore a new application of ICA in MR image analysis where MR images are of major interest. Instead of working on functional information by temporal signals provided by fMRI as most ongoing ICA-based research does, we take rather different approach by considering a set of MR images acquired by different pulse sequences specified by three magnetic resonance parameters, spin-lattice (T1), spin-spin (T2) and dual echo-echo proton density (PD) as a 3-dimeniosnal (3D) image cube where a pulse sequence is represented by a parameter in a third dimension. As a result, an MR image pixel is actually a column vector, each component of which represents an image pixel in an MR image which is specified by a particular pulse sequence. In order for ICA to be used as an image analysis technique, we interpret the number of pulse sequences used in MR acquisition, denoted by L and substances such as blood, fatness, white matter (WM), grey matter (GM), cerebral spinal fluid (CSF), tumors etc. as signal sources to be separated, denoted by p. Interestingly, in this case that the L is generally less than p. As a consequence, the problems of this type to be solved is an under-determined system with $L < p$ where the ICA must deal with an over-complete representation of a mixed model. This is completely opposite to most ICA-based approaches used in fMRI. Therefore, unlike the under-complete ICA which has more dimensions than it requires for blind signal separation, i.e., $L > p$, the over-complete ICA has more signal sources needed to be separated than dimensions, i.e., $p > L$. Under such circumstance, a dimension expansion (DE) is required for the over-complete ICA to make a mixing matrix a square matrix as opposed to the under-complete ICA which requires DR for the same purpose. Unfortunately, there are many techniques available for DR, but not for DE. In order to cope with this problem, a DE technique recently developed for multispectral image classification in [2] is explored in this paper for the over-complete ICA. It expands dimensions by creating new nonlinearly correlated images with original MR images. These newly generated images combined with the original set of MR images provide sufficient number of images required for the ICA to perform blind source separation where the ICA to be used in this paper is the FastICA developed by Hyvarinen and Oja in [3]. In order to demonstrate the utility of our proposed ICA-DE approach in MR image analysis, a set of real brain MR images is used to illustrate MR image classification in practical applications.

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II. INDEPENDENT COMPONENT ANALYSIS

Independent component analysis (ICA) has received consider interest in recent years because of its versatile applications ranging from blind source separation, channel equalization to speech recognition and functional magnetic resonance imaging [1].

The ICA implements a linear mixing model which can be described by

$$\mathbf{x} = \mathbf{A}\mathbf{s} \quad (1)$$

where $\mathbf{x}$ is an L-dimensional mixture source column vector, $\mathbf{A}$ is an $L \times p$ mixing matrix and $\mathbf{s}$ is a p-dimensional column vector with p independent signal sources needed to be demixed from the $\mathbf{x}$. Three important assumptions are made on the mixing model specified by Eq. (1). One is the prior knowledge of p, the number of signal sources to be separated. Another is statistical independency among the p signal sources. A third is no more than one signal source to be Gaussian due to the fact that a linear mixture of Gaussian sources is still Gaussian. Obviously, an idealistic case for Eq. (1) is $L = p$ which can be uniquely solved by matrix inversion provided the mixing matrix is of full rank. Unfortunately, in many applications, this is generally not the case. It has been widely studied for the case of $L > p$ where Eq. (1) has more dimensions than signal sources needed to be separated. In this case, the ICA is referred to as under-complete ICA which has less basis vectors used to solve an over-determined system which does not generally have a solution. Under such circumstance, a dimensionality reduction is generally required to make the mixing matrix A a square matrix with $L = p$. One commonly used approach is principal components analysis (PCA). By contrast, when $L < p$ occurs in Eq. (1), the ICA has more signal sources than the data dimensionality in which case the ICA is referred to as over-complete since it has more basis vectors, p than dimensions L. In this situation, the ICA intends to solve an under-determined system where an infinite number of solutions are available. Interestingly, the problem of this type has received little attention in the past because in most applications of ICA the data dimensionality L in Eq. (1) is number of sample collected to be used for demixing which is usually greater than the number of signal sources to be demixing, a case that can be always satisfied by collecting a sufficient number of samples. However, if the data dimensionality is interpreted otherwise such as multispectral image with L being the number of spectral channels used for data acquisition and p being the number of substances needed to be classified. In these applications, the ICA to be used must be over-complete in which case a dimension expansion (DE) is required to make a mixing matrix a square matrix as opposed to the under-complete ICA which requires dimensionality reduction (DR) for the same purpose. To our best knowledge, the concept of DE has not been explored for over-complete ICA in the literature. In the following section, an approach to DE will be discussed.

When the ICA is implemented, it generally assumes that the mixing matrix is a square matrix. Once this has been done, the ICA is readily applied. In this paper, the FastICA algorithm developed by Hyvarinen and Oja [3] will be used for demixing of signal sources. In doing so, we assume that an image cube has size of $M \times N \times L$ pixels where L is the number of spectral bands and MN is the size of each spectral band image. The image cube can then be represented by a data matrix $\mathbf{X}$ of size $L \times MN$ with L rows and MN columns. In other words, each row in the data matrix $\mathbf{X}$ is specified by a particular spectral band image. As a result, a total of L ICs can be generated by the FastICA for image analysis. Details discussion of ICA can be found in [1].

III. DIMENSIONALITY EXPANSION

As noted, for over-complete ICA, the number of the sources is greater than the number of signal sources, $p > L$. In this case, the mixing matrix is not square. So, the general ICA approach which assumes a square mixing matrix cannot be applied. Recently, a Band Generation Process (BGP) developed in [2] for dimensionality expansion can be used for this purpose. In order to take advantage of hyperspectyral imaging techniques which requires hundreds of spectral bands to be applied to multispectral with only tens of spectral bands, the BGP allows one to expand multispectral images by including new additional band images which can be obtained by finding non-linearly correlated band images with the original set of multispectral band images. The idea of BGP arises from the fact that a second-order random process is generally specified by its first-order and second-order statistics. If the original bands are considered as the first-order statistics images, a set of second-order statistics bands can be generated by finding non-linear correlation between bands. Such non-linearly correlated images provide useful second-order statistics information that is missing in the original bands. Several second-order statistics, including autocorrelation, cross-correlation, and nonlinear correlation can be used to create such nonlinearly correlated images as follows.

Let $\{B_i\}_{i=1}^{L}$ be the set of all original bands. Several methods to create second-order correlated and nonlinear correlated bands via BGP can be described as follows.

A) Second-order correlated bands:

1. $\{B_i^2\}_{i=1}^{L}$ is the set of auto-correlated bands.
2. $\{B_i B_j\}_{i=1, j=1, j \neq i}^{L}$ is the set of cross-correlated bands.

B) Nonlinear correlated bands:

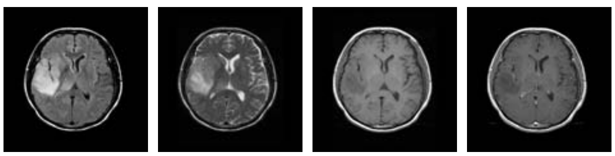
1. $\{\sqrt{B_i}\}_{i=1}^{L}$ is the set of bands stretched out by the square root.

2. $\{\sqrt{B_i B_j}\}_{i=1,j=1,j\neq i}^{L}$.

The above BGP dimensionality expansion techniques will be used to resolve the issue of the over-complete ICA in insufficient number of bands.

IV. EXPERIMENTS

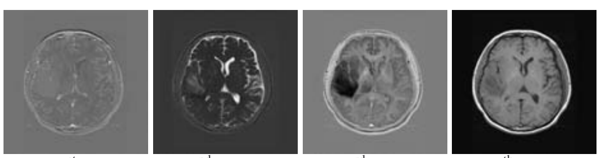
In this section four real brain MR images shown in Fig. 1 were used for experiments to demonstrate the utility of the BGP in DE for performance evaluation. As shown in Band 1 and Band 2, there is a brain tumor shown at the left center which is of major interest.



Band 1 Band 2 Band 3 Band 4

Figure 1. MRI images in different bands

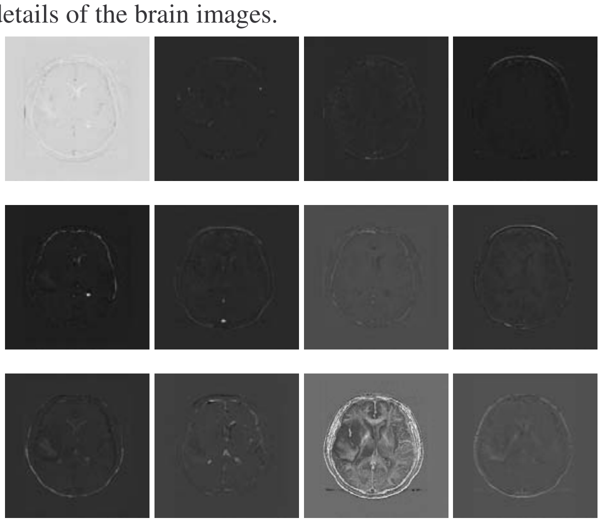
If the FastICA was directly applied to the 4-band image cube in Fig. 1 without the BGP, the four produced independent components (ICs) are shown in Fig. 2 where the tumor is shown in the 3rd IC.



1st IC 2nd IC 3rd IC 4th IC

Figure 2. Four ICs produced by the FastICA without the BGP

However, if the BGP using the DE techniques described by A and B, a total of 20 new images, 4 for A-1, 6 for A-2, 4 for B-1 and 6 for B-2 were generated. These 20 new additional images combined with 4 original band images resulted in 24 band images to be processed by the FastICA. Fig. 3 shows the 24 FastICA-generated ICs, each of which seemed to extract information about certain details of the brain images.



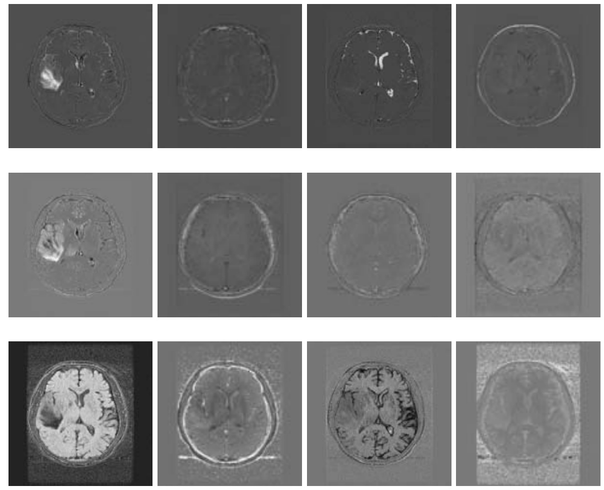
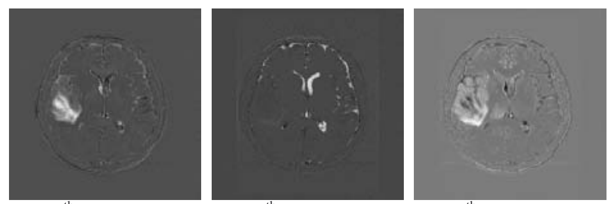
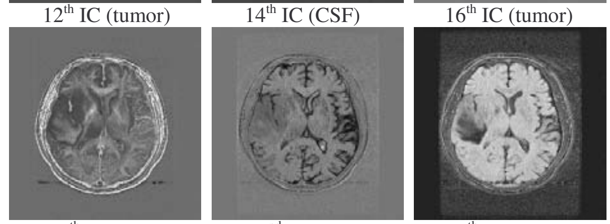


Figure 3. 24 ICs generated by the FastICA

However, by examining the 24 ICs in Fig. 3 carefully, there are 6 ICs among the 24 ICs that represent major information about the brain and they are shown in Fig. 4.



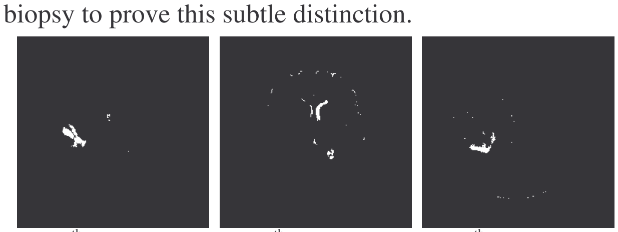
12th IC (tumor) 14th IC (CSF) 16th IC (tumor)



20th IC (skull) 22nd IC (GM) 24th IC (WM)

Figure 4. 6 ICs representing crucial information about the brain

In particular, the tumor was extracted in two separate individual ICs, 12th and 16th ICs due to the fact that the tumor tissue in the 16th IC may be mixed by its background signatures. In this case, it was considered as a different signature from the tumor signature in the 12th IC. Additionally, the CSF, GM and WM were also extracted in 14th, 22nd and 24th ICs. Since the images in Fig. 4 are gray scale, Fig. 5 shows their binary images resulting from thresholding for detection performance. As shown, the tumor in the 12th and the 16th ICs were completely separated. This implied that the tissue in the 16th may be actually not tumor at all. Unfortunately, there was no biopsy to prove this subtle distinction.



12th IC (tumor) 14th IC (CSF) 16th IC (tumor)

Figure 5. Binary images resulting from threshoplding images in Fig. 4

For the purpose of visualization, the 6 brain tissues extracted in Fig. 4 were mapped into a color map in Fig. 6 where the 6 brain tissues were assigned by different colors according to the vertical bar next to its color map for reference.

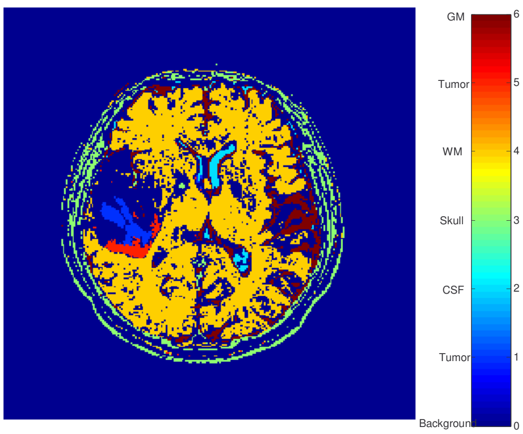


Figure 6. Color map of the 6 extracted brain tissues

As we can see from Fig. 6, this color map provides better visual understanding of 6 brain tissues classification results. Comparing results in Fig. 4 to results in Fig. 2, it obviously shows that the over-complete ICA with the BGP performed significantly better than that without the BGP. This simple experiments provided evidence that the over-complete ICA in junction with a DE improved the over-complete ICA alone.

V. CONCLUSIONS

This paper presents a new method to implement the over-complete ICA for MR image analysis. Since there are no sufficient number of MR band images to perform ICA in demixing signal sources, a technique, called band generation process (BGP) previously developed for multispectral image analysis is further used to expand data dimensionality by including new additional images that are non-linearly correlated with the original MR and images. The experimental results demonstrate that in MR image analysis the over-complete ICA with DE offers advantages over the over-complete ICA without DE.

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